

## Data Fitting

### Exercise 4

The data given in the table was entered into the file `yukawa01.txt`. The script to do the fitting is `Exercise04.py`. It reads the data file, and does the fit. The fit returns the two parameters  $r'$  and  $g^2$ .

We must use error propagation formulas to compute the uncertainties in  $m$  and  $g$  from  $r'$  and  $g^2$ . Recall that the general error propagation formula for determining the uncertainty  $\delta f$  in some variable  $f(x)$  is

$$\delta f = \left| \frac{\partial f}{\partial x} \right| \delta x, \quad (1)$$

where  $\delta x$  is the uncertainties in  $x$ . In this case, since  $m = \frac{\hbar}{cr'}$ ,

$$\delta m = \left| \frac{\partial}{\partial r'} \left( \frac{\hbar}{r'c} \right) \right| \delta r' = \frac{\hbar}{c} \frac{\delta r'}{r'^2} = \frac{\hbar c}{c^2} \frac{\delta r'}{r'^2}. \quad (2)$$

To compute  $\delta g$  we must treat  $(g^2)$  as the variable  $x$  in equation 1, then

$$\delta g = \left| \frac{\partial \sqrt{(g^2)}}{\partial (g^2)} \right| \delta (g^2) = \frac{1}{2\sqrt{(g^2)}} \delta (g^2). \quad (3)$$

These equations are used in `Exercise04.py` to compute the uncertainties in  $m$  and  $g$ . Running `Exercise04.py` gave the following results:

```
In [31]: run Exercise04.py
mass of pion = 150.42 +/- 18.7628 MeV/c^2
coupling constant = 62.7632 +/- 9.52932 (MeV fm)^0.5
reduced chi^2 = 0.837342
goodness-of-fit = 0.473121
g2/hbarc = 19.9656 +/- 6.06276
```

The reduced  $\chi^2$  of 0.83 and the goodness-of-fit probability of 0.47 indicate a good fit as does looking at a plot of the fit with the data. The best fit values of  $m = 150 \pm 19$  MeV/c<sup>2</sup> and  $g = 63 \pm 10$  (MeV fm)<sup>1/2</sup> agree with accepted values of  $m = 139.6$  MeV/c<sup>2</sup> and  $g \approx 54$  (MeV fm)<sup>1/2</sup>.

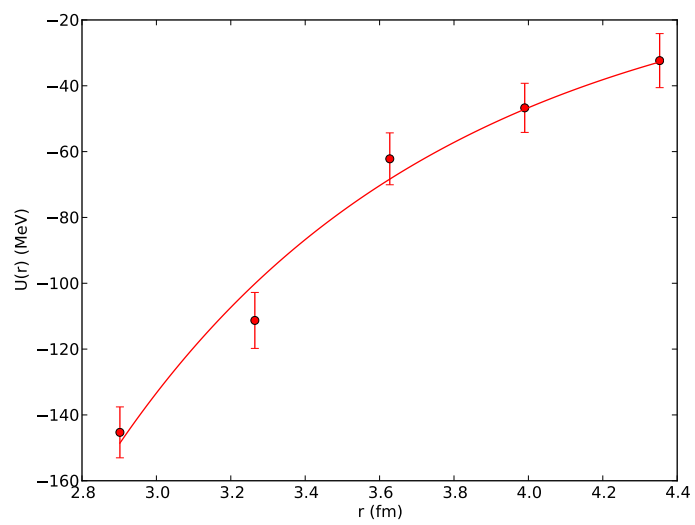


Figure 1: Plot of the data and fit.